Examples 4–38 through 4–41 illustrate the fundamental counting rule.

**Example 4–38**

**Tossing a Coin and Rolling a Die**

A coin is tossed and a die is rolled. Find the number of outcomes for the sequence of events.

![Tree Diagram](image)

**Solution**

Since the coin can land either heads up or tails up and since the die can land with any one of six numbers showing face up, there are $2 \cdot 6 = 12$ possibilities. A tree diagram can also be drawn for the sequence of events. See Figure 4–8.

**Example 4–39**

**Types of Paint**

A paint manufacturer wishes to manufacture several different paints. The categories include:

- **Color**: Red, blue, white, black, green, brown, yellow
- **Type**: Latex, oil
- **Texture**: Flat, semigloss, high gloss
- **Use**: Outdoor, indoor

How many different kinds of paint can be made if you can select one color, one type, one texture, and one use?

**Solution**

You can choose one color and one type and one texture and one use. Since there are 7 color choices, 2 type choices, 3 texture choices, and 2 use choices, the total number of possible different paints is

<table>
<thead>
<tr>
<th>Color</th>
<th>Type</th>
<th>Texture</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

$7 \cdot 2 \cdot 3 \cdot 2 = 84$
Example 4-40  Distribution of Blood Types

There are four blood types, A, B, AB, and O. Blood can also be Rh+ and Rh-. Finally, a blood donor can be classified as either male or female. How many different ways can a donor have his or her blood labeled?

Solution

Since there are 4 possibilities for blood type, 2 possibilities for Rh factor, and 2 possibilities for the gender of the donor, there are 4 \cdot 2 \cdot 2, or 16, different classification categories, as shown.

<table>
<thead>
<tr>
<th>Blood type</th>
<th>Rh</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

A tree diagram for the events is shown in Figure 4-9.

When determining the number of different possibilities of a sequence of events, you must know whether repetitions are permissible.

Example 4-41  Identification Cards

The manager of a department store chain wishes to make four-digit identification cards for her employees. How many different cards can be made if she uses the digits 1, 2, 3, 4, 5, and 6 and repetitions are permitted?
Solution
Since there are 4 spaces to fill on each card and there are 6 choices for each space, the total number of cards that can be made is $6 \cdot 6 \cdot 6 \cdot 6 = 1296$.

Now, what if repetitions are not permitted? For Example 4–41, the first digit can be chosen in 6 ways. But the second digit can be chosen in only 5 ways, since there are only five digits left, etc. Thus, the solution is

$$6 \cdot 5 \cdot 4 \cdot 3 = 360$$

The same situation occurs when one is drawing balls from an urn or cards from a deck. If the ball or card is replaced before the next one is selected, then repetitions are permitted, since the same one can be selected again. But if the selected ball or card is not replaced, then repetitions are not permitted, since the same ball or card cannot be selected the second time.

These examples illustrate the fundamental counting rule. In summary: If repetitions are permitted, then the numbers stay the same going from left to right. If repetitions are not permitted, then the numbers decrease by 1 for each place left to right.

Two other rules that can be used to determine the total number of possibilities of a sequence of events are the permutation rule and the combination rule.

Factorial Notation
These rules use factorial notation. The factorial notation uses the exclamation point.

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$
$$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

To use the formulas in the permutation and combination rules, a special definition of $0!$ is needed. $0! = 1$.

Factorial Formulas
For any counting number $n$

$$n! = n(n - 1)(n - 2) \cdots 1$$
$$0! = 1$$

Permutations

A permutation is an arrangement of $n$ objects in a specific order.

Examples 4–42 and 4–43 illustrate permutations.

Example 4–42 Business Location
Suppose a business owner has a choice of 5 locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the 5 locations?
Solution

There are

\[ 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \]

different possible rankings. The reason is that she has 5 choices for the first location, 4 choices for the second location, 3 choices for the third location, etc.

In Example 4–42 all objects were used up. But what happens when not all objects are used up? The answer to this question is given in Example 4–43.

Example 4–43

Business Location

Suppose the business owner in Example 4–42 wishes to rank only the top 3 of the 5 locations. How many different ways can she rank them?

Solution

Using the fundamental counting rule, she can select any one of the 5 for first choice, then any one of the remaining 4 locations for her second choice, and finally, any one of the remaining locations for her third choice, as shown.

<table>
<thead>
<tr>
<th>First choice</th>
<th>Second choice</th>
<th>Third choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ 5 \cdot 4 \cdot 3 = 60 \]

The solutions in Examples 4–42 and 4–43 are permutations.

Objective 6

Find the number of ways that \( r \) objects can be selected from \( n \) objects, using the permutation rule.

Permutation Rule

The arrangement of \( n \) objects in a specific order using \( r \) objects at a time is called a permutation of \( n \) objects taking \( r \) objects at a time. It is written as \( _nP_r \), and the formula is

\[ _nP_r = \frac{n!}{(n-r)!} \]

The notation \( _nP_r \) is used for permutations.

\[ _5P_4 = \frac{6!}{(6-4)!} \quad \text{or} \quad \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360 \]

Although Examples 4–42 and 4–43 were solved by the multiplication rule, they can now be solved by the permutation rule.

In Example 4–42, 5 locations were taken and then arranged in order; hence,

\[ _5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 120 \]

(Recall that \( 0! = 1 \).)
In Example 4-43, 3 locations were selected from 5 locations, so \( n = 5 \) and \( r = 3 \); hence,

\[
_sP_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60
\]

Examples 4-44 and 4-45 illustrate the permutation rule.

**Example 4-44**

**Television Ads**

The advertising director for a television show has 7 ads to use on the program. If she selects 1 of them for the opening of the show, 1 for the middle of the show, and 1 for the ending of the show, how many possible ways can this be accomplished?

**Solution**

Since order is important, the solution is

\[
_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210
\]

Hence, there would be 210 ways to show 3 ads.

**Example 4-45**

**School Musical Plays**

A school musical director can select 2 musical plays to present next year. One will be presented in the fall, and one will be presented in the spring. If she has 9 to pick from, how many different possibilities are there?

**Solution**

Order is important since one play can be presented in the fall and the other play in the spring.

\[
_9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7!}{7!} = 72
\]

There are 72 different possibilities.

**Combinations**

Suppose a dress designer wishes to select two colors of material to design a new dress, and she has on hand four colors. How many different possibilities can there be in this situation?

This type of problem differs from previous ones in that the order of selection is not important. That is, if the designer selects yellow and red, this selection is the same as the selection red and yellow. This type of selection is called a combination. The difference between a permutation and a combination is that in a combination, the order or arrangement of the objects is not important; by contrast, order is important in a permutation. Example 4-46 illustrates this difference.

A selection of distinct objects without regard to order is called a combination.