Example 5–19

Survey on Fear of Being Home Alone at Night

*Public Opinion* reported that 5% of Americans are afraid of being alone in a house at night. If a random sample of 20 Americans is selected, find these probabilities by using the binomial table.

a. There are exactly 5 people in the sample who are afraid of being alone at night.

b. There are at most 3 people in the sample who are afraid of being alone at night.

c. There are at least 3 people in the sample who are afraid of being alone at night.

Source: *100% American* by Daniel Evan Weiss.

**Solution**

a. \( n = 20, p = 0.05, \) and \( X = 5. \) From the table, we get 0.002.

b. \( n = 20 \) and \( p = 0.05. \) “At most 3 people” means 0, 1, or 2, or 3.

Hence, the solution is

\[
P(X \leq 3) = P(0) + P(1) + P(2) + P(3) = 0.358 + 0.377 + 0.189 + 0.060 = 0.984
\]

c. \( n = 20 \) and \( p = 0.05. \) “At least 3 people” means 3, 4, 5, . . . , 20. This problem can best be solved by finding \( P(0) + P(1) + P(2) \) and subtracting from 1.

\[
P(0) + P(1) + P(2) = 0.358 + 0.377 + 0.189 = 0.924
\]

\[
P(X \geq 3) = 1 - 0.924 = 0.076
\]

Example 5–20

Driving While Intoxicated

A report from the Secretary of Health and Human Services stated that 70% of single-vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 15 single-vehicle traffic fatalities that occur at night on a weekend is selected, find the probability that exactly 12 involve a driver who is intoxicated.

Source: *100% American* by Daniel Evan Weiss.

**Solution**

Now, \( n = 15, p = 0.70, \) and \( X = 12. \) From Table B, \( P(12) = 0.170. \) Hence, the probability is 0.17.

Remember that in the use of the binomial distribution, the outcomes must be independent. For example, in the selection of components from a batch to be tested, each component must be replaced before the next one is selected. Otherwise, the outcomes are not independent. However, a dilemma arises because there is a chance that the same component could be selected again. This situation can be avoided by not replacing the component and using a distribution called the hypergeometric distribution to calculate the probabilities. The hypergeometric distribution is beyond the scope of this book. Note that when the population is large and the sample is small, the binomial probabilities can be shown to be nearly the same as the corresponding hypergeometric probabilities.

**Objective 4**

Find the mean, variance, and standard deviation for the variable of a binomial distribution.

**Mean, Variance, and Standard Deviation for the Binomial Distribution**

The mean, variance, and standard deviation of a variable that has the binomial distribution can be found by using the following formulas.

- Mean: \( \mu = np \)
- Variance: \( \sigma^2 = npq \)
- Standard deviation: \( \sigma = \sqrt{npq} \)
These formulas are algebraically equivalent to the formulas for the mean, variance, and standard deviation of the variables for probability distributions, but because they are for variables of the binomial distribution, they have been simplified by using algebra. The algebraic derivation is omitted here, but their equivalence is shown in Example 5–21.

**Example 5–21**

**Tossing a Coin**

A coin is tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.

**Solution**

With the formulas for the binomial distribution and \( n = 4 \), \( p = \frac{1}{2} \), and \( q = \frac{1}{2} \), the results are

\[
\begin{align*}
\mu &= n \cdot p = 4 \cdot \frac{1}{2} = 2 \\
\sigma^2 &= n \cdot p \cdot q = 4 \cdot \frac{1}{2} \cdot \frac{1}{2} = 1 \\
\sigma &= \sqrt{1} = 1
\end{align*}
\]

From Example 5–21, when four coins are tossed many, many times, the average of the number of heads that appear is 2, and the standard deviation of the number of heads is 1. Note that these are theoretical values.

As stated previously, this problem can be solved by using the formulas for expected value. The distribution is shown.

<table>
<thead>
<tr>
<th>No. of heads ( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability ( P(X) )</td>
<td>( \frac{1}{16} )</td>
<td>( \frac{4}{16} )</td>
<td>( \frac{6}{16} )</td>
<td>( \frac{4}{16} )</td>
<td>( \frac{1}{16} )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\mu &= E(X) = \sum X \cdot P(X) = 0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{4}{16} + 4 \cdot \frac{1}{16} = \frac{16}{16} = 2 \\
\sigma^2 &= \sum X^2 \cdot P(X) - \mu^2 \\
&= 0^2 \cdot \frac{1}{16} + 1^2 \cdot \frac{4}{16} + 2^2 \cdot \frac{6}{16} + 3^2 \cdot \frac{4}{16} + 4^2 \cdot \frac{1}{16} - 2^2 = \frac{68}{16} - 4 = 1 \\
\sigma &= \sqrt{1} = 1
\end{align*}
\]

Hence, the simplified binomial formulas give the same results.

**Example 5–22**

**Rolling a Die**

A die is rolled 480 times. Find the mean, variance, and standard deviation of the number of 3s that will be rolled.

**Solution**

This is a binomial experiment since getting a 3 is a success and not getting a 3 is considered a failure.

Hence, \( n = 480 \), \( p = \frac{1}{6} \), and \( q = \frac{5}{6} \).

\[
\begin{align*}
\mu &= n \cdot p = 480 \cdot \frac{1}{6} = 80 \\
\sigma^2 &= n \cdot p \cdot q = 480 \cdot \frac{1}{6} \cdot \frac{5}{6} = 66.67 \\
\sigma &= \sqrt{66.67} = 8.16
\end{align*}
\]
Example 5–23  

Likelihood of Twins

The Statistical Bulletin published by Metropolitan Life Insurance Co. reported that 2% of all American births result in twins. If a random sample of 8000 births is taken, find the mean, variance, and standard deviation of the number of births that would result in twins.

Source: 100% American by Daniel Evan Weiss.

Solution

This is a binomial situation, since a birth can result in either twins or not twins (i.e., two outcomes).

\[
\mu = np = (8000)(0.02) = 160
\]

\[
\sigma^2 = np(1-p) = (8000)(0.02)(0.98) = 156.8
\]

\[
\sigma = \sqrt{np(1-p)} = \sqrt{156.8} = 12.5
\]

For the sample, the average number of births that would result in twins is 160, the variance is 156.8, or 157, and the standard deviation is 12.5, or 13 if rounded.

Applying the Concepts 5–3

Unsanitary Restaurants

Health officials routinely check sanitary conditions of restaurants. Assume you visit a popular tourist spot and read in the newspaper that in 3 out of every 7 restaurants checked, there were unsatisfactory health conditions found. Assuming you are planning to eat out 10 times while you are there on vacation, answer the following questions.

1. How likely is it that you will eat at three restaurants with unsanitary conditions?
2. How likely is it that you will eat at four or five restaurants with unsanitary conditions?
3. Explain how you would compute the probability of eating in at least one restaurant with unsanitary conditions. Could you use the complement to solve this problem?
4. What is the most likely number to occur in this experiment?
5. How variable will the data be around the most likely number?
6. How do you know that this is a binomial distribution?
7. If it is a binomial distribution, does that mean that the likelihood of a success is always 50% since there are only two possible outcomes?

Check your answers by using the following computer-generated table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00371</td>
<td>0.00371</td>
</tr>
<tr>
<td>1</td>
<td>0.02784</td>
<td>0.03155</td>
</tr>
<tr>
<td>2</td>
<td>0.09396</td>
<td>0.12552</td>
</tr>
<tr>
<td>3</td>
<td>0.18793</td>
<td>0.31344</td>
</tr>
<tr>
<td>4</td>
<td>0.24665</td>
<td>0.56009</td>
</tr>
<tr>
<td>5</td>
<td>0.22199</td>
<td>0.78208</td>
</tr>
<tr>
<td>6</td>
<td>0.13874</td>
<td>0.92082</td>
</tr>
<tr>
<td>7</td>
<td>0.05946</td>
<td>0.98028</td>
</tr>
<tr>
<td>8</td>
<td>0.01672</td>
<td>0.99700</td>
</tr>
<tr>
<td>9</td>
<td>0.00279</td>
<td>0.99979</td>
</tr>
<tr>
<td>10</td>
<td>0.00021</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

See page 296 for the answers.
Exercises 5–3

1. Which of the following are binomial experiments or can be reduced to binomial experiments?
   a. Surveying 100 people to determine if they like Sudsy Soap
   b. Tossing a coin 100 times to see how many heads occur
   c. Drawing a card with replacement from a deck and getting a heart
   d. Asking 1000 people which brand of cigarettes they smoke
   e. Testing four different brands of aspirin to see which brands are effective
   f. Testing one brand of aspirin by using 10 people to determine whether it is effective
   g. Asking 100 people if they smoke
   h. Checking 1000 applicants to see whether they were admitted to White Oak College
   i. Surveying 300 prisoners to see how many different crimes they were convicted of
   j. Surveying 300 prisoners to see whether this is their first offense

2. (ans) Compute the probability of $X$ successes, using Table B in Appendix C.
   a. $n = 2, p = 0.30, X = 1$
   b. $n = 4, p = 0.60, X = 3$
   c. $n = 5, p = 0.10, X = 0$
   d. $n = 10, p = 0.40, X = 4$
   e. $n = 12, p = 0.90, X = 2$
   f. $n = 15, p = 0.80, X = 12$
   g. $n = 17, p = 0.05, X = 0$
   h. $n = 20, p = 0.50, X = 10$
   i. $n = 16, p = 0.20, X = 3$

3. Compute the probability of $X$ successes, using the binomial formula.
   a. $n = 6, X = 3, p = 0.03$
   b. $n = 4, X = 2, p = 0.18$
   c. $n = 5, X = 3, p = 0.63$
   d. $n = 9, X = 0, p = 0.42$
   e. $n = 10, X = 5, p = 0.37$

For Exercises 4 through 13, assume all variables are binomial. (Note: If values are not found in Table B of Appendix C, use the binomial formula.)

4. Guidance Missile System  A missile guidance system has five fail-safe components. The probability of each failing is 0.05. Find these probabilities.
   a. Exactly 2 will fail.
   b. More than 2 will fail.
   c. All will fail.
   d. Compare the answers for parts a, b, and c, and explain why these results are reasonable.

5. True/False Exam A student takes a 20-question, true/false exam and guesses on each question. Find the probability of passing if the lowest passing grade is 15 correct out of 20. Would you consider this event likely to occur? Explain your answer.

6. Multiple-Choice Exam A student takes a 20-question, multiple-choice exam with five choices for each question and guesses on each question. Find the probability of guessing at least 15 out of 20 correctly. Would you consider this event likely or unlikely to occur? Explain your answer.

7. Driving to Work Alone It is reported that 77% of workers aged 16 and over drive to work alone. Choose 8 workers at random. Find the probability that
   a. All drive to work alone
   b. More than one-half drive to work alone
   c. Exactly 3 drive to work alone
Source: www.factfinder.census.gov

8. High School Dropouts Approximately 10.3% of American high school students drop out of school before graduation. Choose 10 students entering high school at random. Find the probability that
   a. No more than two drop out
   b. At least 6 graduate
   c. All 10 stay in school and graduate
Source: www.infoplease.com

9. Survey on Concern for Criminals In a survey, 3 of 4 students said the courts show "too much concern" for criminals. Find the probability that at most 3 out of 7 randomly selected students will agree with this statement.
Source: Harper's Index.

10. Labor Force Couples The percentage of couples where both parties are in the labor force is 52.1. Choose 5 couples at random. Find the probability that
    a. None of the couples have both persons working
    b. More than 3 of the couples have both persons in the labor force
    c. Fewer than 2 of the couples have both parties working
Source: www.bls.gov

11. College Education and Business World Success R. H. Bruskin Associates Market Research found that 40% of Americans do not think that having a college education is important to succeed in the business world. If a random sample of five Americans is selected, find these probabilities.
    a. Exactly 2 people will agree with that statement.
    b. At most 3 people will agree with that statement.
    c. At least 2 people will agree with that statement.
    d. Fewer than 3 people will agree with that statement.
Source: 100% American by Daniel Evans Weiss.

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